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COMMENTS ON GUST RESPONSE CONSTRAINED OPTIMIZATION

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ABSTRACT

Recent research pertaining to optimum structural design with probabilistic constraints is reviewed. The limitations and complexities introduced in the design as a result of the transition from deterministic to probabilistic constraints are underscored. A concise development of the theoretical aspects of optimum design of aircraft structures subjected to random wind loads is presented and suggestions for future research are offered. An emphasis is placed on the incorporation of recent developments in fracture mechanics in the design constraints.

INTRODUCTION

An overwhelming majority of recent developments in optimum structural design have dealt primarily with the minimum weight design of a statically loaded structure. Structural design with constraints on the dynamic response characteristics introduces an additional degree of complexity and has been the subject of recent research. A random vibration environment necessitates a reformulation of the optimization problem and presents significant new problems that are the subject of this paper.

Optimum design of structural systems with random parameters and probabilistic constraints is a physically realistic problem. Ground excitation during an earthquake or unsteady wind shears are examples of random loads. A similar situation exists for an airplane flying into patches of storm or nonstorm turbulence. There are two levels of difficulty that can be identified in this problem:

(a) A systematic description of the random loads and the choice of a statistical process that would allow the computation of the dynamic response parameters of interest

(b) The interpretation of these parameters for the optimum design problem (this would include the formulation of constraints that would minimize the conservativeness in the design but would still be computationally viable)

A considerable body of literature exists in the civil engineering discipline that deals specifically with the description of random loads. A power spectral density description is perhaps the most common approach to the problem wherein the frequency spectrum of ground motion or air pressure distribution is specified. Reference 1 reviews the subject in some detail. There were considerably fewer publications pertaining to probabilistic or reliability based optimum design. References 2-4 are indicative of attempts at optimization in a nondeterministic environment for simplistic structural configurations.

The primary focus of the present paper is to review the state-of-art approaches for probabilistic design in aerospace applications. Research efforts in this area are typified by References 5-8. The deficient areas are defined and new methodology presently under study is outlined. The numerical results obtained under this study will be presented in a separate publication.

AIR TURBULENCE MODELS

Measurements of various air turbulence samples indicate both a time and a spatial variation. A rapid penetration of the gust field justifies assumption of freezing the gust in time (ref. 9). This assumption would be invalid in rotorcraft applications and for air vehicles that have a significant hover mode. In most response calculations, the spatial variation of the gust field along a spanwise direction is neglected and only a variation in the flight direction is taken into consideration. This one-dimensional model (fig. 1) needs to be reassessed for light, high aspect ratio airplanes.

Computation of the system response in the frequency domain is more elegant than the time domain solution and is therefore emphasized in this paper. Following this approach a turbulence field is typically characterized by the gust velocity power spectral density (PSD) distribution shown in figure 2.

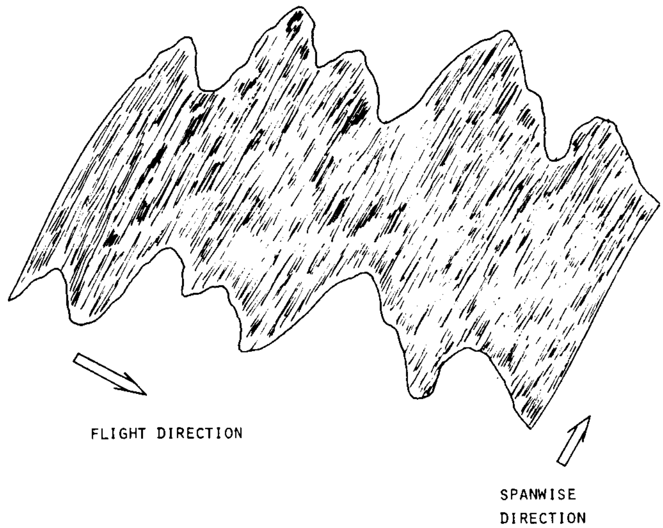

Figure 1.- One-dimensional turbulence model.

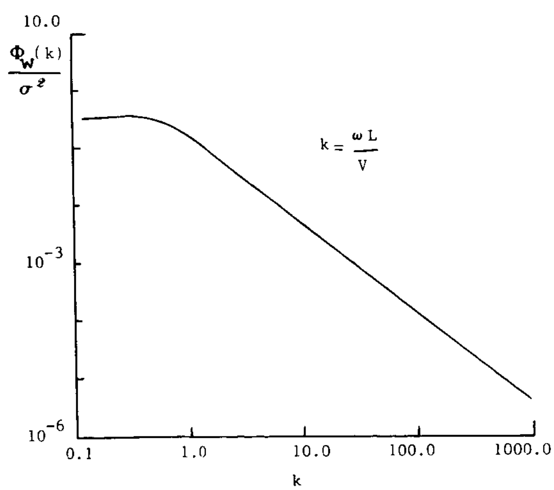

Figure 2.- von Karman power spectra.

The power spectral density function is representative of the variation of the mean square values of the gust velocities with frequency and is established on the basis of a stationary, isotropic, homogeneous, one-dimensional gust field characterized by a Gaussian probability distribution. The power spectral density distribution for the gust intensity is used in conjunction with the response admittance functions to compute the linear response of the system to the gust loading (Figure 3). The von Karman spectra given by

$$\phi(\Omega) = \frac{\sigma^2 L}{\pi} \frac{1 + \frac{8}{3}(1.339L\Omega)^2}{[1 + (1.339L\Omega)^2]^{11/6}}$$

provides a good fit to recorded turbulence data. Here, σ is the rms turbulence intensity and 'L' is the "scale of turbulence" which is representative of a spatial distance over which no correlation exists in the gust intensities. Flight measurements indicate some disagreement with the stationarity assumption. This lack of agreement is in the higher turbulence load levels where the computed response underestimates the actual response. This suggests a modification in the Gaussian model for turbulence.

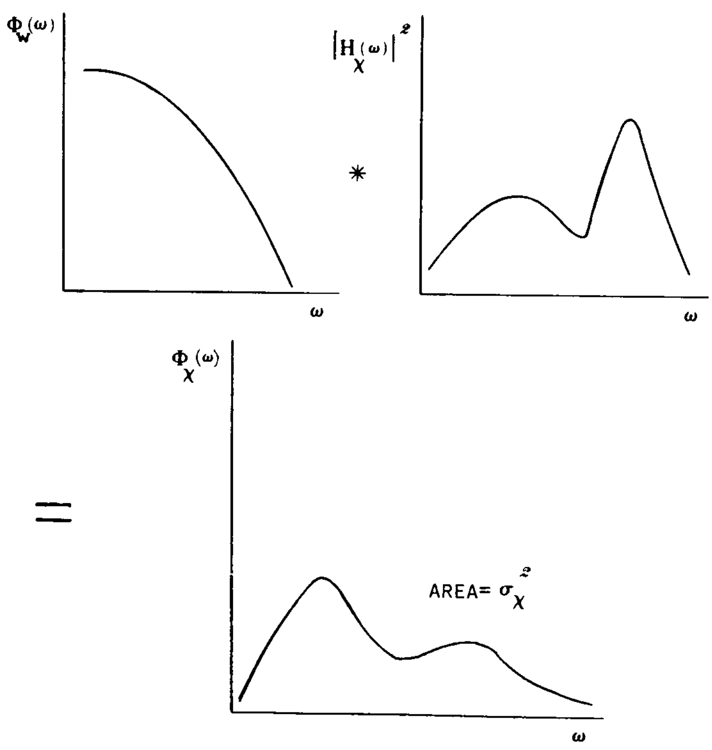

Figure 3.- Linear response analysis in the frequency domain.

GUST RESPONSE ANALYSIS

A modal response approach is customarily preferred for the dynamic analysis of large structural systems. A finite number of elastic modes and dominant rigid body modes are used to model the structural deformations. In terms of the displacement vector w, the system equations of motion are written as

$$\left(-\omega^2 [M] + [K] - [A]\right)\{w\} = \{G\}$$

where [M] and [K] are the mass and stiffness matrices, [A] is the matrix of loads due to oscillation at frequency ω, and {G} is the force coefficient array per unit gust velocity. The displacements {w} can be represented approximately by the superposition of the characteristic modes:

$$\{w\} = [\Phi]\ \{q\}$$

where $[\Phi]$ is the matrix of 'm' normalized eigenmodes and {q} is an m-dimensional vector of modal participation factors. The system equation is rewritten as

$$(-\omega^2[I] + [\diagdown \omega_m^2 \diagdown] - [\bar{A}])\ \{q\} = \{\bar{G}\}$$

Here ω_m^2 is the m-th natural frequency and arrays $[\bar{A}]$ and $\{\bar{G}\}$ are defined as

$$[\bar{A}] = [\Phi]^T\ [A][\Phi]$$

$$\{\bar{G}\} = [\Phi]^T \{G\}$$

The above set of simultaneous algebraic equations is solved for a range of reduced frequencies of interest. The forces and the stresses are computed in terms of the displacements.

THE STRUCTURAL OPTIMIZATION PROBLEM

Structural optimization problem in a stochastic environment must account for the random variation in design parameters in addition to the random dynamic loads. The problem formulation is stated as

$$\text{Minimize} \quad F(\bar{d})$$

$$\text{Subject to} \quad P[\bigcup_{\substack{i=1 \\ a<t<b}}^{k} \{R_i(\bar{w}(\bar{d},t)) > r_i\}] < [p_f]$$

$$R_j(\bar{d}) < r_j$$

$$d_i^L < d_i < d_i^u$$

Here $F(\bar{d})$ is typically the structural weight, $\bar{d}$ is the vector of design variables and $\bar{w}$ is the time varying dynamic response. R_i is the response function with a deterministic or random bound specified by r_i, and $[p_f]$ represents the upper bound on the probability of failure. The design variables have prescribed lower and upper bounds.

In the event that $R_i(\bar{w}\ (\bar{d},t))$ is a stationary random process, the constraint can be rewritten in a deterministic, time independent manner (ref. 10).

$$P[\bigcup_{\substack{i=1 \\ a<t<b}}^{k} \{R_i(\bar{w}(\bar{d},t)) > r_i\}] \approx \sum_{i=1}^{k} q_i(\bar{d}) < (p_f)$$

where, for R and r both conforming to a normal distribution and exhibiting statistical independence,

$$q(\bar{d}) = \frac{1}{\sqrt{2\pi}} \int_{\mu_r - \mu_R / \sqrt{\sigma_r^2 + \sigma_R^2}}^{\infty} e^{-u^2/2} du$$

where the μ's and σ's denote the mean and rms values, respectively. The solution to the ensuing deterministic problem can be approached by any standard nonlinear programming strategy. While this approach may perform well for simplistic situations involving single response quantities, gust response design poses significant new problems. The power spectral density approach used in aircraft gust design leads to the rms values of individual loads such as shear, bending moment and torsion at various points within the structure. However, the statistical nature of these loads conceals information regarding their combination characteristic (sign and magnitude), which is very important from the standpoint of design. A constant probability of combination criterion has been suggested to circumvent this problem. The normal probability distribution density function (fig. 4) for two variables, x and y, is given as

$$p(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho_{xy}^2}} \exp\left[-\frac{1}{2(1-\rho_{xy}^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho_{xy}\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right]$$

This equation represents ellipses in planes parallel to the x-y plane and the infinite load combinations on the boundary of the ellipse have an equal probability of occurrence. The shape and orientation of the ellipse depend upon the mean and mean square values of x and y and on a quantity ρ_{xy}, referred to as the correlation coefficient. In an attempt to define a finite number of dominant load combinations, Stauffer and Hoblitt (ref. 7) propose a technique to circumscribe the ellipse by an octagon (fig. 5) and use the eight vertices of the octagon as critical load combinations in the structural design. This procedure would be intractable if a higher number of equally probable loads were to be combined.

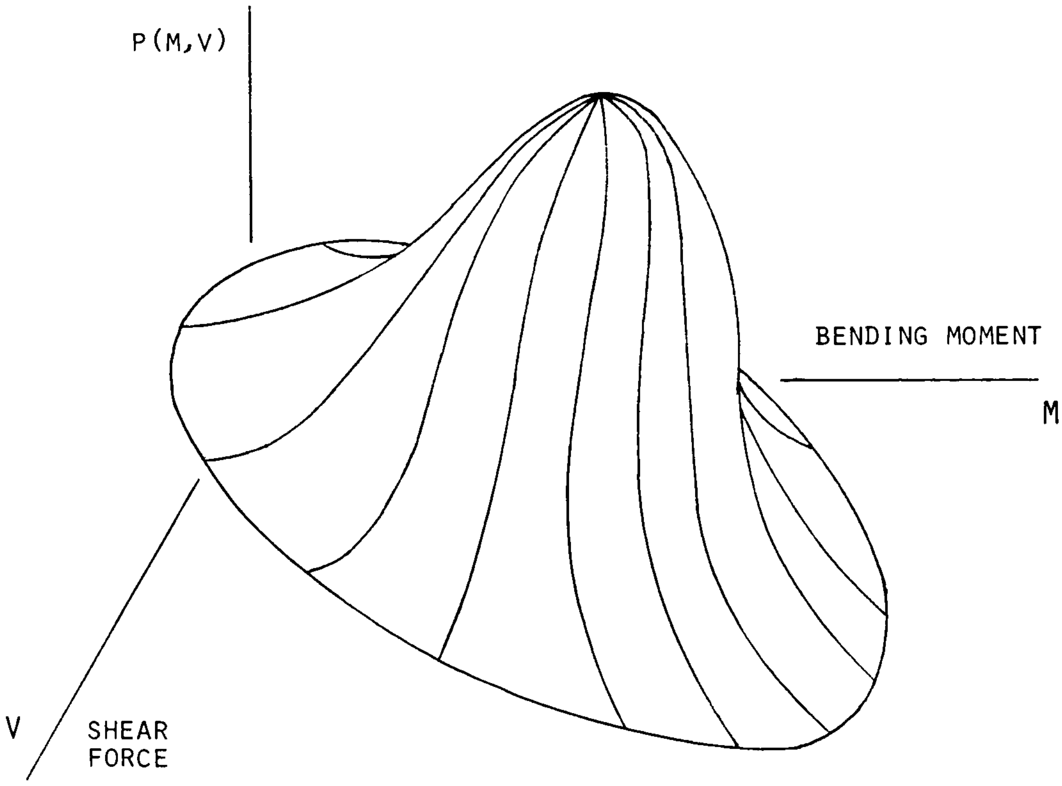

Figure 4.- Probability density.

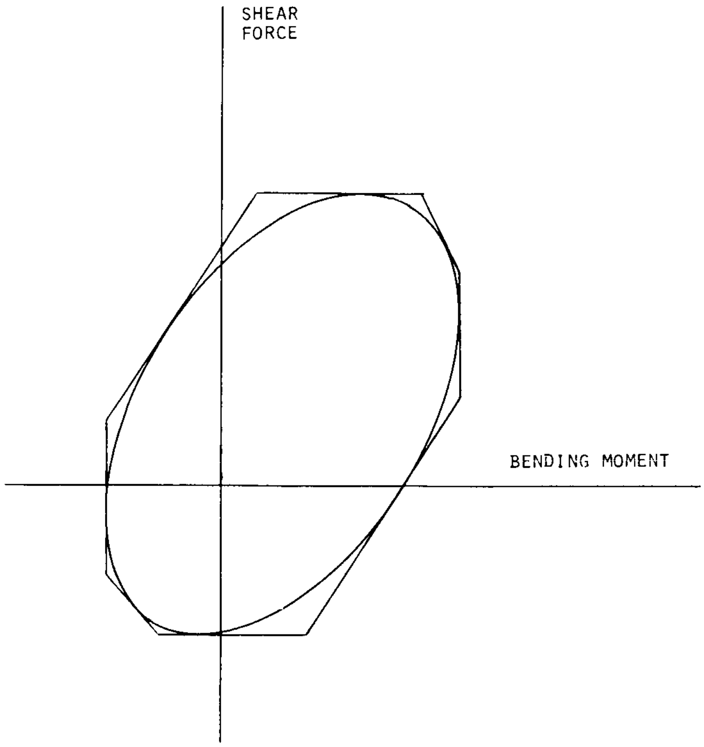

Figure 5.- Equal probability of load combination.

Gross and Sobieski (ref. 6) suggest an alternate procedure which would be applicable to a general multivariate normal density distribution. This approach discretizes the equal probability curve/hypersurface into a finite number of design-load-combination conditions. In an optimization framework this would translate into a very large number of constraints, a situation that is countered by recourse to the "cumulative constraint" idea which permits folding these constraints into a single representative measure.

In both these strategies the correlation coefficient ρ_{xy} plays a key role. This quantity in defined as follows

$$\rho_{xy} = \frac{1}{\bar{A}_x \bar{A}_y} \int_0^\infty \phi_w(\omega) \left[H_x(\omega)_{real} * H_y(\omega)_{real} + H_x(\omega)_{imag} * H_y(\omega)_{imag} \right] d\omega$$

where $\phi_w(\omega)$ is the gust power spectral density; H_x and H_y are the frequency response functions for the load quantities x and y; $\bar{A}_x$ and $\bar{A}_y$ are the ratios of the design rms loads σ_x and σ_y to the design rms gust intensity σ_w, respectively. The value of ρ_{xy} varies between +1 and -1 with $\pm$ 1 representing complete statistical dependence and a value of 0 representing complete statistical independence. Since this quantity varies with the change in the stiffness and mass distribution, it is imperative to define approximations to this coefficient which would be computationally less cumbersome.

DIRECT APPROACH

The approach presently under study computes the power spectral density for a combined stress function and constrains the rms values of this function by prescribed bounds. Consider a combined stress constraint of the form

$$R(\sigma_x, \sigma_y, \tau_{xy}) < R_{all}$$

where R is a stress interaction curve and is constrained by an upper bound R_{all}. By the methods of an earlier section, the generalized force vector can be written as

$$\{F\} = ([A] - \omega^2[M])\{w\} + \{G\}$$

The quantities σ_x, σ_y and τ_{xy} are functions of the shear V, torque T, bending moment M, and the material properties MP.

The constraint can be expressed as

$$R(V,T,M,MP) < R_{all}$$

Furthermore, the forces and moments can be expressed in terms of the force vector {F} through a transfer matrix based on the structural geometry:

$$\{V\} = [T_1]\{F\}$$

$$\{M\} = [T_2]\{F\}$$

$$\{T\} = [T_3]\{F\}$$

The admittance for the composite response function R, denoted here as H_R, can be computed over a range of frequencies of interest and the rms response value evaluated as

$$\sigma_R^2 = \int_0^\infty |H_R|^2 \, \Phi_w(\omega) \, d\omega$$

It is obvious that the procedure requires considerable numerical resources and an important emphasis in the ongoing study would be to establish guidelines to minimize this investment.

RELIABILITY CONSTRAINTS IN PROBABILISTIC DESIGN

In a stochastic excitation environment, there are two logical failure criteria of comparable significance:

(1) Single excursion failure which corresponds to an overstress in any one cycle of loading

(2) Fatigue failure that results from a gradual degradation in the structural strength due to cyclic loading

Johnson (ref. 11) formulates constraints for a response function x(t) that is stationary and has a Gaussian distribution. An assumption that large values of x(t) arrive independently leads to a Poisson probability function for the number of times n that a large magnitude $\bar{X}$ is exceeded in time t. If T_s denotes the desired life for the structure and $\bar{X}_s$ is the specified value that the response cannot exceed, the constraint to guard against a single excursion failure is written as

$$g_{s.e} = 1 - \frac{T_s}{\pi} \frac{\sigma_{\dot{x}}}{\sigma_x} \exp\left(- \bar{X}_s^{\,2}/2\sigma_x^2\right) > 0$$

For a fatigue failure analysis, the classical Palmgren-Miner theory provides an estimate of the rate of fatigue damage and the constraint is formulated as

$$g_{f \cdot f} = 1 - \frac{\sigma_{\dot{x}} L_f}{\pi c} (2)^{\frac{b-2}{2}} \sigma_x^{\,b-1} \;\Gamma\left(\frac{b+2}{2}\right) > 0$$

Here, L_f is the desired fatigue life; σ_x and $\sigma_{\dot{x}}$ are the rms value of the response and the response rate and b and c are constants obtained from an empirical relation

$$N(x) = \frac{c}{x^b}$$

where N(x) is the number of cycles to failure at stress level x. The above formulation is based on an analysis in the frequency domain and is applicable to response functions taken one at a time or to a combined response function.

RELIABILITY CONSTRAINTS FROM THE FRACTURE MECHANICS STANDPOINT

Reference 12 presents a comprehensive discussion on recent advances in fracture mechanics. Empirical relationships express the degradation in the material in terms of fault or crack propagation rates and provide better estimates of the useful life of the structure than the Palmgren-Miner theory. Incorporation of these ideas in the design constraints is a principal focus of the ongoing study. Consider the cracked specimen shown in figure 6. The differences between various cracked components is expressed in terms of a stress intensity factor, k. This factor describes the stress field around a crack tip and is functionally dependent on the stress σ and the crack geometry denoted by 'a'. There is a critical stress intensity constant k_c for a given material. For the crack shown in Fig. 6, this functional relationship can be expressed in terms of the range of stress intensity factor Δk and the range of stress variation $\Delta\sigma$:

$$\Delta k = \Delta\sigma \sqrt{\pi a}$$

The factor $\pi^{1/2}$ changes as the crack size increases in comparison to the characteristic width of the specimen.

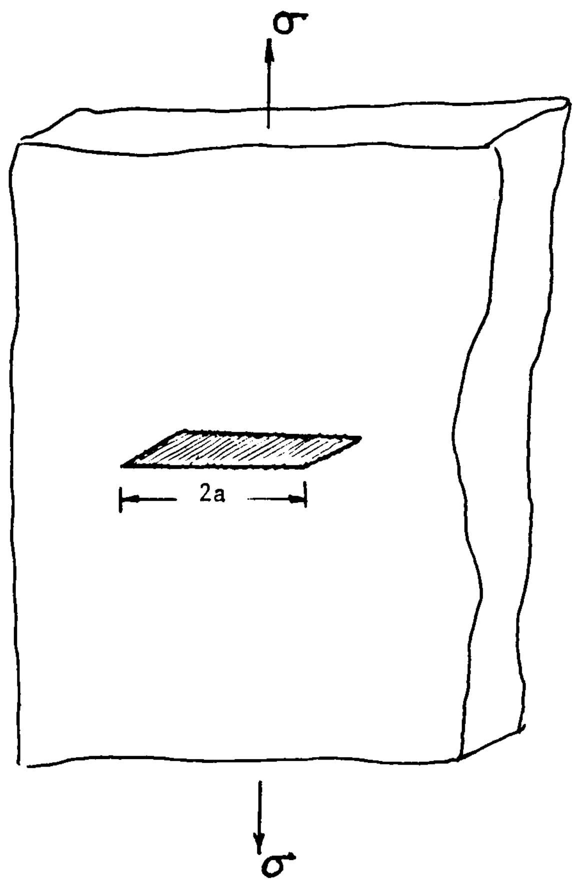

Figure 6.- Cracked material specimen.

The crack propogation rate per cycle of load is given as

$$\frac{da}{dn} = c\Delta k^{b}$$

where c and b are constants dependent on the material of the specimen. For a value of b = 2 (steel) one can write an expression for the crack size after t flight hours as

$$a(t) = a_o\ e^{c\pi\Delta\sigma^2 N_o t}$$

where N_o is the number of gust loads per flight hour; $(\Delta\sigma)^2$ is the mean square value of a Gaussian response function σ. The Griffith Irwin equation* indicates the residual strength in a material with crack size 'a' in terms of the critical stress intensity constant k_c:

$$k_c = R\sqrt{\frac{\pi a}{2}}$$

The time to reach a critical crack size a_c is obtained as**

$$t_c = \frac{1}{c\pi\Delta\sigma^2 N_o} \ln \frac{a_c}{a_o}$$

At t_f flight hours after t_c, the crack size is

$$a_f = a_c e^{c\pi\Delta\sigma^2 N_o t_f}$$

and the residual strength at this time is obtained as

$$R_f = R_c\ e^{c\pi\Delta\sigma^2 N_o t_f}$$

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* Equation is valid for determinate structures only and modifications are necessary for redundant structures.

**Note that a_c generally corresponds to the material ultimate strength, R_c.

Design constraints can thus be formulated to require t_c to be greater than a specified time T_c

$$g_1 \equiv 1 - \frac{T_c}{\ln \frac{a_c}{a_o}} \; c\pi\Delta\sigma^2 N_o$$

Another constraint could be formulated that required the residual strength to be above a certain bound ($R_f/R_c > K_1$), T_f flight hours after the critical crack size is reached. Such a constraint would be prescribed from a maintenance schedule:

$$g_2 \equiv 1 - \frac{T_f}{\ln K_1} \; c\pi\Delta\sigma^2 N_o > 0$$

SUMMARY

The major finding of the present survey can be outlined as follows.

(a) The frequency domain analysis provides an elegant solution strategy to the gust response problem. However, the lack of agreement in measured data with assumptions such as stationarity, normality and the one-dimensional variation in gust velocity dictate the need for reassessing the turbulence modeling.

(b) The phase information regarding load combinations is suppressed in a frequency domain solution. This is problematic when combined stress constraints are prescribed in the design. Strategies to circumvent these problems are stated.

(c) The definition of realistic constraints for the optimum design is addressed from the standpoint of recent developments in fracture mechanics.

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